

On the Shape of the First Collapsed Objects

S.F. Shandarin, A.L. Melott, K. McDavitt, J.L. Pauls, and J. Tinker

Department of Physics and Astronomy, University of Kansas, Lawrence, Kansas 66045

(February 1, 2008)

Abstract

Since the early seventies, there was a conjecture that the first collapse of a selfgravitating dust-like medium (appropriate approximation for nonbaryonic dark matter) results in the formation of a “pancake” object, that is a thin surface. The conjecture has been based on the Zel’dovich approximate solution of the nonlinear gravitational instability of a generic smooth density perturbation. Recent works cast doubt on the Zel’dovich conjecture, suggesting that the first collapse might be point-like or filament-like rather than pancake-like. Our N -body simulations show first pancake collapse. We can reject with 97% confidence the Bayesian prior that the other kinds of collapse are more or equally probable.

95.30Sf, 04.90+e, 98.80Dr

Typeset using REVTeX

I. INTRODUCTION

In the evolution of gravitational clustering in the expanding universe, it has gradually been recognized that the first collapse is usually anisotropic. This has important consequences on all scales, from the formation of stars, galaxies, or superclusters. Although it may be visible today only in superclusters, which are just collapsing now, it has consequences for the formation of all objects by gravity, assuming mathematically generic initial conditions without special symmetry and neglecting pressure gradients.

A schematic history of the question follows: The known exact solution obtained for the spherically symmetric, non-rotating, pressure-free case, (e.g. [1]) predicts two types of collapse from rest. If the initial density is monotonically decreasing then the first collapse is point-like toward the origin. However, if the initial density is non-monotonic then the first collapse is shell-like. Considering a uniform, non-rotating, pressure-free spheroid Lin et. al. [2] found that it collapses either toward a disk or a spindle depending on whether it is oblate or prolate at the initial time.

Zel'dovich [3] proposed an approximation for a generic initial perturbation which predicts that the first collapsed objects have a pancake-like shape. Gurevich and Zybin [4] revisited the issue and concluded that the nondissipative gravitational collapse of a generic perturbation results in the formation of a stationary dynamical structure with a point-like singularity at its core $\rho \propto r^{-24/13}$. Recently, Bertschinger [5] and Bertschinger and Jain [6] proposed a purely local gravitational instability solution based on General Relativity which implied prolate collapse to filaments comes first. This would have strong implications for star, galaxy, and supercluster formation, if it is also true in weak gravitational fields inside the horizon. Kofman and Pogosyan [7] and Bertschinger and Hamilton [8] showed that this solution had neglected certain terms of the same order as others included in it which may be justified in ultrarelativistic cases, but not in the Newtonian limit. The difference between the collapse in a dust-like matter in Newtonian and ultrarelativistic cases was stressed in Zel'dovich and Novikov [9]; see also Matarrese, Pantano and Saez [10]. This

provides a renewed justification for the neo-Newtonian approach generally used for studying low-amplitude cosmological perturbations inside the horizon.

However, as noted by Bertschinger and Hamilton [8] it does not resolve the question whether pancakes or filaments form first. Although the Zel'dovich [3] approximation (ZA) predicts pancakes, this approximation is not exact in three dimensions. It is known, for example, that collapse in nonlinear gravitational clustering simulations proceeds faster than ZA predicts. It is therefore important to determine whether the quasi-two-dimensional structures predicted by ZA really occur.

One should distinguish between two statements we might make: (1) The first collapse is always pancake-like. (2) The first collapse is usually pancake-like but could be filament-like in some special cases. In this paper, we present evidence for the second (weaker) statement based on numerical simulations.

The initial conditions we set up are of a generic type, which means that a smooth small arbitrary perturbation does not change qualitatively the type of initial condition in any sense.

II. COMPUTATIONS

We examined an ensemble of five N -body simulations on a 128^3 Particle-Mesh gravitational clustering code with periodic boundary conditions. Initial conditions are impressed by the now-standard use of a random number generator to create choices of phase and Gaussian-distributed amplitude for various Fourier components of the initial density fluctuations, and the motions in response to these. At the very low amplitudes the ZA we used and Eulerian linear perturbation theory of the growing mode are essentially indistinguishable. We also stress that using shot noise with a dying mode component [11] or a logarithmic distribution of modes [12] has not made noticeable differences. Further details on simulation methods can be found in [13]. The initial conditions corresponding to the growing mode were constructed in four realizations with initial fluctuations of wavenumber 1 through $\sqrt{3}$

in units of the fundamental mode of the box. Thus, the minimum wavelength present in initial conditions is 74 mesh units. Although we wished to examine the first collapse in detail, a smaller upper bound on wavenumber would cause alignment with coordinate axes. An additional simulation with initial wavenumber range 1 through 3 (minimum wavelength 43 mesh units) was performed as a check (#1 in Table I). We found nothing special in this case. All simulations were started with *rms* density fluctuation $\sigma \sim 0.03 - 0.04$ in order to allow time (an expansion factor of ~ 17) for transients to die out and full growing mode including nonlinear effects to establish itself. Two simulations (#2 and #4), as a check, started with half the initial amplitude and ran for twice the expansion factor. We found no particular difference between runs with different amplitudes. At the initial stage the density perturbations look like ordinary three-dimensional smooth Gaussian fields. The structures shown below are resulted from nonlinear growth of the density fluctuations due to gravitational instability.

All previous studies of the collapse of a smooth perturbation suggest that the trajectories of the particles are smooth prior to the shell-crossing (see e.g. [1,9]). We have found no evidence against this assumption. We also have found no evidence that the particles might bunch up thickly without undergoing following shell-crossing. Thus we follow Zel'dovich and define the first collapsed objects as the regions where the first shell-crossing occurs. Formally this definition does not assume that the particles undergone shell-crossing form a gravitationally bound object, though it is likely at the later stages.

We stopped the simulations after the *first* shell crossings. Our timesteps are very strictly constrained so that the *fastest* particle could travel 0.4 grid cell (out of 128) in a single timestep. All particles were tagged which had local shell crossing (as determined by whether the local volume element had gone negative). Thousands of particles typically shell crossed for the first time in a single step. We then examined the distribution of these particles. It is worth stressing that the particles in question show the regions between caustics and do not represent well the density distribution. They were all highly anisotropic and resembled surfaces rather than lines. One was ribbon-like but still essentially very flat. We will

illustrate this with multiple pictures from one simulation; other simulations look similar.

Figure 1 shows three orientations of a typical surface (#5 in Table I) viewed along the three eigenvectors of the initial deformation tensor toward the middle of the surface. Figures 1a and 1b suggest finite thickness (~ 1 to 6) but this is because the surface is curved (bowl-like). In Figure 2 we show two cross sections of this surface to indicate its thinness. Figure 1c shows the pancake region face on. The reader should not be misled by the little rows of particles which are the usual result of the standard “quiet start” with particles on a slightly deformed cubic lattice.

Figure 2a shows a cross section perpendicular to the z -axis and Figure 2b shows a cross section perpendicular to the y -axis. Both cross sections are very thin. They suggest that the real thickness of the region (\sim the distance between caustics) is of order one mesh unit, while the diameters (size in y and z directions as seen in Fig. 1a) are about 37 and 17 mesh units. From this we conclude that the shape of the region is pancake-like with approximate ratios 1:17:37, rather than filament-like. We looked closely at many more additional thin slices and concluded that the actual maximum thickness was always < 0.8 , in agreement with the timestep constraint. The other dimensions were much larger, as can be easily seen.

Catastrophe theory suggests that the diameters of a pancake grow as $\sim (t - t_c)^{1/2}$ and its thickness (defined as the distance between caustics) as $\sim (t - t_c)^{3/2}$, therefore the ratio of the thickness to the diameter is proportional to $\sim (t - t_c)$ at small $t - t_c$ (here t_c is the time of the formation of the first singularity) [14]. Also the diameters are not equal to each other in a generic case. In our simulation we plot Figures 1 and 2 after a small but finite time from the first local crossing (the first “singularity” to the accuracy of the simulation) and therefore expect small but finite thickness of the pancake.

In contrast to Figure 2, Figure 3 shows *all* particles in thin slices orthogonal to the principal axes of the initial deformation tensor at the largest eigenvalue. One can easily see the difference between the density distributions (Fig. 3) and the shape of the collapsed region (Fig. 2). All the statements about the shapes of the first collapsed regions derived from ZA refer to the shapes of collapsed regions (Fig. 1 and 2) which may be similar but not

the same as the density distributions (Fig. 3) especially if the resolution is not sufficiently good. We believe that it is worth keeping in mind this difference while analyzing the results of N -body simulations.

III. CONCLUSION

Our simulations suggest that (to the limit of their accuracy) the first stage of collapse of a generic gravitational system is usually to a thin sheet as suggested by ZA. (Obviously we cannot say anything about the evolution of shapes between the last “uncollapsed” and the first “collapsed” stages, but we stress that our timesteps are shorter relative to the characteristic formation time of the structures under consideration than any simulations to date.) This should be taken into account in all gravitational instability theory from star formation through large-scale clustering. Superclusters, now experiencing their first collapse, should include sheetlike structures. Filaments (another type of generic structure [14]) may be easier to see due to their higher density contrast and possible gas cooling effects (J.P. Ostriker, personal communication), but our results indicate they will be second generation objects formed by flows inside sheets.

In the presence of small-scale perturbations in the initial spectrum (which is the most likely case in cosmology) these pancake-like structures are not as smooth as the pancakes discussed in this paper. As we mentioned before, there is a theoretical question concerning the type of the first collapse in a dust-like medium. Our results should not be interpreted as totally excluding the possibility of the first collapse to filament-like structures. It is well known from second order perturbation theory that the rate of collapse along one principal axis depends on the rates of collapse along the other principal axes. In principal, it may change the type of the collapse in some cases. On the other hand, the general solution with the maximal number (eight) of physically arbitrary functions of three variables in a dust like medium suggests gravitational collapse is pancake-like [15,16]. In Katz *et al* [17] it is stated that “the first objects form in filaments from almost two-dimensional collapses in agreement

with the approximate analytic theory of Bertschinger and Jain,” which appears to contradict our results. We did not investigate all options for Gaussian initial conditions. Our initial conditions were particular random realizations of Gaussian initial conditions, with formally k^{-1} power spectrum of density fluctuations in the range of $k_f \leq k \leq \sqrt{3}k_f$ (or $3k_f$ in one case). But we would like to stress that they were of a mathematically generic type.

We have presented in detail the results of the simulation of one pancake. However, we totally studied five realizations of the initial conditions. All showed similar pancake-like structures (sometimes elongated); see Table 1 where we list the thickness, width and length for the first collapsed objects in our first five realizations. We find neither a single filament-like collapse in our simulations, (filaments would be expected on the basis of the hypothesis of Bertschinger and collaborators[5,6]) nor point-like collapse [4]. If we use a prior hypothesis that the ZA and HA descriptions of first collapse are equally probable, we can reject this on the basis of our experiments with 97% confidence. Alternatively, we may assume pancakes and other structures form with some probabilities and try to estimate that probability. A sequence of five pancakes would be more probable than the sum of all other sequences’ probabilities if the a priori probability of a pancake were 87%. We note that our objects are all smaller (much smaller in thickness) than the minimum wavelength in the initial perturbations, and thus represent the first generation of collapsed objects. Objects formed on any scale in hierarchical clustering N -body simulations, such as those of Katz *et al.* [17], are larger than the Nyquist wavelength of the initial spectrum, and therefore a later generation and irrelevant to the question studied here. However, we do comment that such simulations might be expected to show one-dimensional collapse on the objects where the things are just becoming mildly nonlinear. Recently, observational evidence has appeared to suggest there are sheetlike neutral hydrogen clouds at moderate redshift [19]. Quantitative evidence for pancake-like morphology for such objects (as well as filaments existing at later stages of dynamical evolution) has been found in hierarchical clustering simulations [18]. However, this technique does not measure the distance between caustics, discussed in this paper, and does not take into account the thin bowl-like shape of the

first pancakes. The formation of the filament-like structures as well as compact clumps of higher density contrast than in pancakes, in the frame of ZA was emphasized in Arnold *et al.* [14]. This may explain why pancakes are not easily seen in low *mass* resolution N -body simulations.

IV. ACKNOWLEDGEMENTS

We thank Lev Kofman for a stimulating presentation which inspired this study, and he as well as Phil Baringer, Ed Bertschinger, and Jerry Ostriker provided useful discussions. Research support came from NSF grant AST-9021414 (including Research Experiences for Undergraduates), NASA grant NAGW-3832 and the National Center for Supercomputing Applications. S.F. Shandarin acknowledges a University of Kansas GRF 94 Grant.

REFERENCES

- ¹ P.J.E. Peebles, “The Large-Scale Structure of the Universe” (Princeton University Press, Princeton, 1980) Chap. 19.
- ² C.C. Lin, L. Mestel and F.M. Shu, *Ap.J.* **142**, 1431 (1965).
- ³ Ya.B. Zel’dovich, *A&A* **5**, 84 (1970).
- ⁴ A.A. Gurevich and K.P. Zybin, *Sov.Phys.JETP* **67**, 1 (1988).
- ⁵ E. Bertschinger, *Proc. Cosmic Velocity Fields* eds. F. Bouchet and M. Lachièze-Rey, Edition Frontiers, 1993) p. 137.
- ⁶ E. Bertschinger and B. Jain, *Ap.J.* **431**, 486 (1994).
- ⁷ L.A. Kofman and D. Pogosyan, *Ap.J.* **442**, 30 (1995).
- ⁸ E. Bertschinger and A. Hamilton, *Ap.J.* **435**, 1 (1994).
- ⁹ Ya.B. Zel’dovich and I.D. Novikov, *The Structure and Evolution of the Universe* (The University of Chicago Press, Chicago, 1983), Chap. 19.
- ¹⁰ S. Matarrese, O. Pantano, and D. Saez, *Phys.Rev.Lett.* **72**, 320 (1994).
- ¹¹ J.F. Beacom, K.G. Dominik, A.L. Melott, S.P. Perkins, and S.F. Shandarin, *Ap.J.* **372**, 351 (1991).
- ¹² A. Nusser and A. Dekel, *Ap.J.* **362**, 14 (1990).
- ¹³ A.L. Melott and S.F. Shandarin, *Ap.J.* **410**, 469 (1993).
- ¹⁴ V.I. Arnold, S.F. Shandarin, and Ya.B. Zel’dovich, *Geophys.Astrophys.Fluid Dyn.* **20**, 111, (1982).
- ¹⁵ E.M. Lifshitz, V.V. Sudakov, and I.M. Khalatnikov, *Sov.Phys. JETP*, **13**, 1298 (1961).
- ¹⁶ L.P. Grishchuk, *Sov.Phys. JETP*, **24**, 320 (1967).

- ¹⁷ N. Katz, T. Quinn, E. Bertschinger, and J. Gelb, M.N.R.A.S. in press (1994).
- ¹⁸ A. Babul and G.D. Starkman, Ap.J. **401**, 28 (1992).
- ¹⁹ N. Dinshaw, C.B. Foltz, C.D. Impey, R.J. Weymann, and S.L. Morris, Nature **373**, 223 (1995).

V. FIGURE CAPTIONS

FIG. 1 Three projections of the collapsed points (past the singular stage) orthogonal to three principal axes of the initial deformation tensor.

FIG. 2 Two thin slices (2 mesh units) approximately through the center of the pancake orthogonal to (a) z -axis: $15 \leq z \leq 17$; (b) y -axis $10 \leq y \leq 12$.

FIG. 3 The mass distribution in thin slices orthogonal to three principal axis:

(a) z -axis: $15 \leq z \leq 17$

(b) y -axis: $10 \leq y \leq 12$, and

(c) x -axis: $112 \leq x \leq 114$.

TABLES

TABLE I.

Thickness in mesh units	Width in mesh units	Length in mesh units
0.1	3.5	8
0.5	5	16
0.6	32	37
0.8	7.5	48
0.8	17	37

Information on the objects in the five simulations